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Effect of size on quantized conductance in a dirty quantum wire

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Abstract. Making use of the Zubarev correlation functions, we calculate and discuss the quantized conductance for a Tomonaga–Luttinger (TL) liquid coupled to phonons in a dirty quantum wire. In this system, the electron–phonon (e–p) coupling increases the conductance, while the impurities in the quantum wire reduce the conductance. The quantum wire length satisfies a certain condition, which determines whether e–p scattering will dominate over impurity scattering. However, the conductance is destroyed under certain conditions: namely, there is a size effect.

1. Introduction

In recent years, great progress has been made in the study of quantum transport in all kinds of mesoscopic systems. One of the most remarkable phenomena is the quantized conductance in a point contact. The most important research in conductance theory is to calculate the quantized conductance and the conductance fluctuations.

The quantized conductance Γ is observed in a channel quantum point contact which is equivalent to a very short quantum wire of less than 0.2 μm [1]. Twenty years ago, Landauer [2] proposed that the dc conductance of noninteracting (spinless) electrons in a disordered medium in strictly one dimension is given by $\Gamma = (e^2/2\pi\hbar)|t|^2/|r|^2$, where t and r are the transmission and reflection amplitudes. In one-dimensional electronic systems, the mutual interaction between electrons plays a most important role which leads to Luttinger-liquid behaviour [3]. But in the theory of Landauer, the mutual interaction has been largely ignored. At low temperatures, it was argued that the Coulomb interaction causes deviations of the conductance Γ from $2e^2/h$ in a channel quantum wire. Some authors [4] calculated the conductance under the influence of interacting electrons. But experiments [5, 6] showed that the usual reduction Γ due to electron–electron interactions is not observed in realistic wires which are always coupled to leads where the interaction is screened. In a recent article [7], some authors showed that the conductivity had to be understood as the current response to the macroscopic electronic field and not to the external field, which was pointed out by Izuyama more than 30 years ago shortly after Kubo had presented his theory of linear response. So the Coulomb interaction has no effect on the conductivity and hence the conductance itself. In this paper, the results of our calculations support this.

Some effort has been devoted to investigating the possibility that e–p interactions may be a candidate for $\Delta\Gamma$. It was demonstrated [7] that the e–p coupling causes the conductance Γ to increase. However, in a realistic quantum wire, it is also necessary to consider the influence of impurities on the conductance properties. Impurities can alter the optical and

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electrical properties. In a TL liquid [8, 9], the quantized conductance may be destroyed. To our knowledge, there have to date been no calculations of the conductance with the e–p mechanism in a dirty quantum wire. Even in investigations which considered the influence of impurities only, mostly numerical computations are used and it is difficult to obtain an analytical result. In this paper, we try to give an analytical result for the conductance in a dirty quantum wire with the e–p mechanism.

2. Theoretical model

In this paper we consider a narrow quantum wire with length L_s and width W, and with impurities randomly distributed along the wire. We know that Anderson localization becomes important for samples with short mean free paths and at low temperatures. Here we investigate wires which are relatively cleaner with mean free paths long enough so that the localization effect is of minor importance. We also assume that the carrier density and W are such that there exists only a single subband along the wire. In our model, the Hamiltonian is

$$
\begin{aligned}
H &= H_0 + H_{e-e} + H_p + H_{e-p} + H_{imp} \\
H_0 &= \sum_\alpha \varepsilon_\alpha C_\alpha^+ C_\alpha \\
H_{e-e} &= \tfrac{1}{2} \sum_\lambda \nu(\lambda) \rho_\lambda \rho_{-\lambda} \\
H_p &= \sum_Q \omega_Q a_Q^+ a_Q \\
H_{e-p} &= \sum_{\alpha\beta Q} M_{\alpha\beta}^Q C_\alpha^+ C_\alpha (a_Q + a_{-Q}^+) \\
H_{imp} &= \sum_\lambda u(\lambda) \sum_\tau C_{\lambda+\tau}^+ C_\tau
\end{aligned}
\tag{1}
$$

where ε_α is the kinetic energy of electrons with effective mass m^* and ω_Q is the phonon frequency. a_Q (a_Q^+) is the creation (annihilation) operator of phonons, C_α (C_α^+) is the creation (annihilation) operator of electrons and $M_{\alpha\beta}^Q$ is the e–p coupling matrix element. $\nu(\lambda)$ and $u(\lambda)$ are the one-dimensional Fourier transforms of the Coulomb interaction and the random impurity potential, respectively. The density operator of electrons ρ_λ is defined as $\rho_\lambda = \sum_\tau C_\tau^+ C_{\tau-\lambda}$.

The conductance can be obtained from the density–density correlation function

$$
\chi(q,\omega) = \mathrm{i} \int_0^\infty \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t} \langle [\rho'_q(t), \rho'_{-q}(0)] \rangle_0
\tag{2}
$$

where $\rho'_q = (1/\sqrt{L_s}) \sum_{k\sigma} C_{k\sigma}^+ C_{k-q\sigma}$ with k referring to the plane wave vector in the x direction. The relation of the conductivity $\sigma(q,\omega)$ to $\chi(q,\omega)$ is given through charge and current conservation,

$$
\sigma(q,\omega) = -\mathrm{i} e^2 (\omega/q^2) \chi(q,\omega)
\tag{3}
$$

with

$$
\chi(q,\omega) = -\frac{1}{L_s} \sum_{kk'} \sum_{\sigma\sigma'} \langle\langle C_{k\sigma}^+ C_{k-q\sigma} \mid C_{k\sigma}^+ C_{k-q\sigma} \rangle\rangle_\omega .
\tag{4}
$$

The conductance $\Gamma(\omega)$ is defined as the spatial average of the conductivity in real space over an interval of length L,

$$\Gamma(\omega) = \langle \sigma(q,\omega)\rangle_L$$
$$\langle A\rangle_L = \frac{1}{L^2}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \mathrm{d}x\,\mathrm{d}x' \int_{-\infty}^{\infty} \frac{\mathrm{d}q}{2\pi}\, \mathrm{e}^{\mathrm{i}q(x-x')} A(q) \tag{5}$$

with the notation $\langle A\rangle_L$ for a function $A(q)$. The Zubarev correlation function in (4) is defined as [10]

$$\langle\langle A \mid B\rangle\rangle_\omega = -\mathrm{i}\int_0^\infty \mathrm{d}t\,\mathrm{e}^{\mathrm{i}\omega t}\langle[A(t), B(0)]\rangle_0 \tag{6}$$

where the expectation value $\langle\rangle_0$ refers to the equilibrium density operator $\exp(-\beta H)/Z$, β is the inverse temperature and $Z = \mathrm{Tr}\exp(-\beta H)$. The correlation function can be obtained from the equations of motion of the Green functions in ω space.

$$\omega\langle\langle A \mid B\rangle\rangle_\omega = \langle[A,B]\rangle_0 + \langle\langle [A,H] \mid B\rangle\rangle_\omega$$
$$\omega\langle\langle A \mid B\rangle\rangle_\omega = \langle[A,B]\rangle_0 - \langle\langle A \mid [B,H]\rangle\rangle_\omega. \tag{7}$$

In the process of calculation, we make an approximation to cut off high-order Green functions. We also consider weak coupling and scattering and take the scattering of phonons and impurities to second order. The resulting analytic expressions are as follows:

$$\begin{aligned}
\langle\langle C_\alpha^+ C_\beta \mid C_\gamma^+ C_\delta\rangle\rangle_\omega = {} & \frac{f_\alpha - f_\beta}{\omega_{\alpha\beta}}\delta_{\alpha\delta}\delta_{\beta\gamma} + \frac{f_\alpha - f_\beta}{\omega_{\alpha\beta}}\frac{f_\delta - f_\gamma}{\omega_{\delta\gamma}}\sum_Q M^Q_{\beta\alpha}M^Q_{\delta\gamma}\left(\frac{1}{\omega-\omega_Q} - \frac{1}{\omega+\omega_Q}\right) \\
& + \frac{1}{\omega_{\alpha\beta}}\frac{f_\delta - f_\gamma}{\omega_{\delta\gamma}}[u(\beta-\gamma)\delta_{\alpha\delta} - u(\delta-\alpha)\delta_{\beta\gamma}] \\
& - \frac{1}{\omega_{\alpha\beta}}\frac{f_\delta - f_\gamma}{\omega_{\delta\gamma}}u(\beta-\gamma)u(\delta-\alpha)\left(\frac{1}{\omega_{\alpha\gamma}} - \frac{1}{\omega_{\delta\beta}}\right) \\
& + \frac{1}{\omega_{\alpha\beta}}\frac{f_\delta - f_\gamma}{\omega_{\delta\gamma}}\sum_\lambda u(\lambda)\left[\frac{u(\beta-\gamma-\lambda)}{\omega_{\alpha\beta-\lambda}}\delta_{\alpha\delta} - \frac{u(\delta-\alpha-\lambda)}{\omega_{\alpha+\lambda\beta}}\delta_{\beta\gamma}\right] \\
& - \frac{f_\alpha - f_\beta}{\omega_{\alpha\beta}}\frac{1}{\omega_{\delta\gamma}}\sum_Q\sum_\lambda u(\lambda)M^Q_{\beta\alpha}M^{-Q}_{\delta\gamma+\lambda}\left(\frac{f_{\delta-\lambda} - f_\gamma}{\omega_{\delta-\lambda\gamma}} - \frac{f_\delta - f_{\gamma+\lambda}}{\omega_{\delta\gamma+\lambda}}\right) \\
& \times\left(\frac{1}{\omega-\omega_Q} - \frac{1}{\omega+\omega_Q}\right) + \frac{1}{\omega_{\alpha\beta}}\frac{f_\delta - f_\gamma}{\omega_{\delta\gamma}}\sum_Q\sum_\lambda u(\lambda)M^Q_{\beta\alpha+\lambda}M^{-Q}_{\delta\gamma} \\
& \times\left(\frac{f_\alpha - f_{\beta-\lambda}}{\omega_{\alpha\beta-\lambda}} - \frac{f_{\alpha+\lambda} - f_\beta}{\omega_{\alpha+\lambda\beta}}\right)\left(\frac{1}{\omega-\omega_Q} - \frac{1}{\omega+\omega_Q}\right).
\end{aligned} \tag{8}$$

In the expression for (8), we use $\omega_Q = \omega_{-Q}$, $v(q) = v(-q)$ and the fact that the momentum matrix element $M^Q_{kk'}$ depends on k and k' only through the difference k-k' for plane waves. Here, we introduce the abbreviation $\omega_{\alpha\beta} = \omega + \varepsilon_\alpha - \varepsilon_\beta$ and $f_\alpha = \{\exp[\beta(\varepsilon_\alpha - \mu) + 1]\}^{-1}$, where μ denotes the chemical potential of the Fermi distribution f. In the process of calculation of the electronic Zubarev correlation function $\langle\langle C_\alpha^+ C_\beta \mid C_\gamma^+ C_\delta\rangle\rangle_\omega$, only the Fermi distribution f occurs. This is consistent with [7]. Making use of (2), (4) and (8), one obtains the concrete expression for the density–density correlation function

$$\chi(q,\omega) = \chi_0(q,\omega) - [\chi_0(q,\omega)]^2\frac{\pi v_F}{2}\gamma_q(\omega) + \chi_1(q,\omega) + \chi_2(q,\omega) - \chi_0(q,\omega)\kappa_q(\omega) \tag{9}$$

where

$$\chi_0(q,\omega) = -\frac{1}{L_s}\sum_{k\sigma}\frac{f_k - f_{k-q}}{\omega + 2kq - q^2} \tag{10}$$

$$\gamma_q(\omega) = \frac{2L_s}{\pi\bar{h}^2 v_F}\sum_Q |M^Q_{kk+q}|^2 \frac{2\omega_q}{\omega^2 - \omega_Q^2}. \tag{11}$$

In (9), $\chi_1(q,\omega)$ and $\chi_2(q,\omega)$ are the contributions of the impurities, and $\kappa_q(\omega)$ is a common interaction term for impurities and phonons. The concrete forms of these terms are determined by (2), (4) and (8). In (11), the expression $\gamma_q(\omega)$ contains the phonon propagator $\sim 1/(\omega^2 - \omega_Q^2)$ but no distribution functions and thus is temperature independent [7].

3. Discussion

In this section, we calculate and discuss in some detail the conductance at $T = 0$ and in the long-wave limit ($q \to 0$) and for the static state ($\omega \to 0$). There are four parts below which consider various possible situations.

(a) When e–p interactions are not considered (matrix element M equals zero) and in the absence of impurities (Fourier translation $u(\lambda)$ equals zero), the density–density correlation function is

$$\chi_0(q,\omega) = -\frac{2v_F q^2}{\pi}\frac{1}{\omega^2 - v_F^2 q^2} \qquad q \to 0 \qquad k_B T \ll \mu. \tag{12}$$

One obtains the conductance

$$\Gamma_0(\omega) = \frac{2e^2}{h}\left(1 + \frac{\mathrm{i}\omega L}{3v_F} + 0(\omega^2)\right) \tag{13}$$

which is the well known result for the conductance of a ballistic one-dimensional channel [2, 7].

(b) When there are no impurities, using the notation

$$\gamma_0 = -\gamma_{q=0}(\omega = 0) = \frac{2L_s}{\pi\bar{h}^2 v_F}\sum_Q |M^Q_{kk}|^2 \frac{2}{\omega_Q} \tag{14}$$

one obtains for the conductance

$$\Gamma(\omega) = \Gamma_0(\omega)(1 + \tfrac{1}{2}\gamma_0). \tag{15}$$

Here, for weak e–p coupling ($\gamma_0 \ll 1$), our calculation is approximately consistent with the result [7] $\Gamma(\omega \to 0) = (2e^2/h)(1-\gamma_0)^{-1/2}$. Moreover, for a realistic wire, γ_0 is larger than zero. Thus at $T = 0$, the e–p coupling leads to an effectively attractive interaction between the electrons which increases the conductance.

(c) When the e–p scattering is weak enough to be neglected, and considering electron–impurity scattering, we obtain

$$\Gamma(\omega \to 0) = \Gamma_0(1 - \tfrac{1}{2}\gamma_1) \tag{16a}$$

where

$$\Gamma_0 = 2e^2/h \qquad \gamma_1 = \frac{8\pi u^2(2k_F) + L_s v_F^2 u^2(0)}{12\pi v_F^4}. \tag{16b}$$

Here, v_F is Fermi velocity. We know from (16) that the scattering terms of moment $2k_F$ have important contributions to the conductance and the scattering of impurities to electrons makes the conductance decrease. The discussion above is consistent with [8]. But, making use of the Zubarev method, we only discuss the situations at $T = 0$ and in the thermodynamic limit, while [8] was based on the Mori formula and discussed at definite temperature. When $\gamma_1 \ll 2$, that is, the length of quantum wire satisfies $L_s \ll L_0$, where $L_0 = 4\pi v_F^{-1}(6v_F^4 - 2u^2(2k_F))u^{-2}(0)$, the quantized conductance can be maintained. This can be realized when the length L_s is short enough and v_F satisfies $v_F \sim (1/3u^2(2k_F))^{1/4}$. In contrast, when $\gamma_1 \to 2$, that is $L_s \to L_0$, the quantized conductance is destroyed by being suppressed.

(d) When we have both impurities and e–p scattering in a quantum wire, the conductance is

$$\Gamma(\omega \to 0) = \Gamma_0[1 + \tfrac{1}{2}(\gamma_0 - \gamma_0')] \tag{17}$$

where

$$\gamma_0' = \gamma_1 + \gamma_2$$
$$\gamma_2 = \frac{4v_F L_s}{\pi^2} \sum_Q \frac{I_Q}{\omega_Q} M_{00}^Q$$
$$I_Q = \sum_{\lambda(\neq 0, \pm k_F, \pm 2k_F)} \frac{u(\lambda)}{\lambda(\lambda + 2k_F)} \ln\left|\frac{(\lambda + k_F)(\lambda - 2k_F)^2}{(\lambda - k_F)(\lambda + 2k_F)^2}\right|(M_{0\lambda}^{-Q} - M_{0\lambda}^Q). \tag{18}$$

We see from (17) that when $\gamma_0 > \gamma_0'$, that is, $L_s > L_1$, where

$$L_1 = \frac{8\pi^2\bar{h}^2u^2(2k_F)}{48\pi v_F^3 \sum_Q |M_{kk}^Q|^2 \frac{1}{\omega_Q} - 48v_F^5\bar{h}^2 \sum_Q \frac{I_Q}{\omega_Q} M_{00}^Q - \pi v_F \bar{h}^2 u^2(0)} \tag{19}$$

the common interactions between phonons and impurities will make the conductance increase. e–p scattering is dominant, which results in an effectively attractive interaction between electrons. On the other hand, for $\gamma_0 < \gamma_0'$, that is, $L_s < L_1$, the common interaction will make the conductance decrease. Thus, the dominance of electron–impurity scattering leads to impulsive interactions between electrons. Under the condition $\gamma_0 - \gamma_0' \ll 2$, we obtain $L_s \ll L_2$, where

$$L_2 = \frac{8\pi^2\bar{h}^2[u^2(2k_F) + 3v_F^4]}{48\pi v_F^3 \sum_Q |M_{kk}^Q|^2 \frac{1}{\omega_Q} - 48v_F^5\bar{h}^2 \sum_Q \frac{I_Q}{\omega_Q} M_{00}^Q - \pi v_F \bar{h}^2 u^2(0)}. \tag{20}$$

Then the quantized conductance can be maintained with only slight deviations. On the other hand, for $\gamma_0 - \gamma_0' \to 2$, that is $L_s \to L_2$, the common interaction of the phonons and impurities will make the quantized conductance disappear.

4. Conclusion

In summary, we have discussed the TL liquid in a quantum wire. Considering the influence of both phonons and impurities, we obtained the expression for the density–density correlation function making use of the equations of motion for the Green functions. From this, we obtained an expression for the conductance and discussed it under four situations. We found that the e–p coupling increases the conductance, while impurity scattering decreases the conductance. When there exist impurities in the quantum wire and we consider also the e–p coupling, there will be a critical value L_1 that has the dimensions of length. When $L_s > L_1$, e–p scattering will be dominant, while for $L_s < L_1$, electron–impurity scattering will have the chief effect, which makes the conductance decrease.

Furthermore, we also gave the condition under which the conductance can be maintained or destroyed.

References

[1] Wharam D A, Thornton T J, Newbury R, Pepper M, Ahmed H, Frost J E F, Hasko D G, Peacock D C, Ritchie D A, and Jones G A 1988 *J. Phys. C: Solid State Phys.* **21** L209
[2] Landauer R 1970 *Phil. Mag.* **21** 863
[3] Haldane F D M 1981 *J. Phys. C: Solid State Phys.* **14** 2585
[4] Apel W and Rice T M 1982 *Phys. Rev.* B **26** 7063
[5] Ponomarenko V V 1995 *Phys. Rev.* B **52** R8666
[6] Safi I and Schulz H 1995 *Phys. Rev.* B **52** R17 040
[7] Brandes T and Kawabata A 1996 *Phys. Rev.* B **54** 4444
[8] Ogata M and Fukuyama H 1993 *Phys. Rev. Lett.* **73** 468
[9] Zhu N J, Guo H and Harris R 1996 *Phys. Rev. Lett.* **77** 1825
[10] Zubarev D N 1960 *Sov. Phys.–Usp.* **3** 320